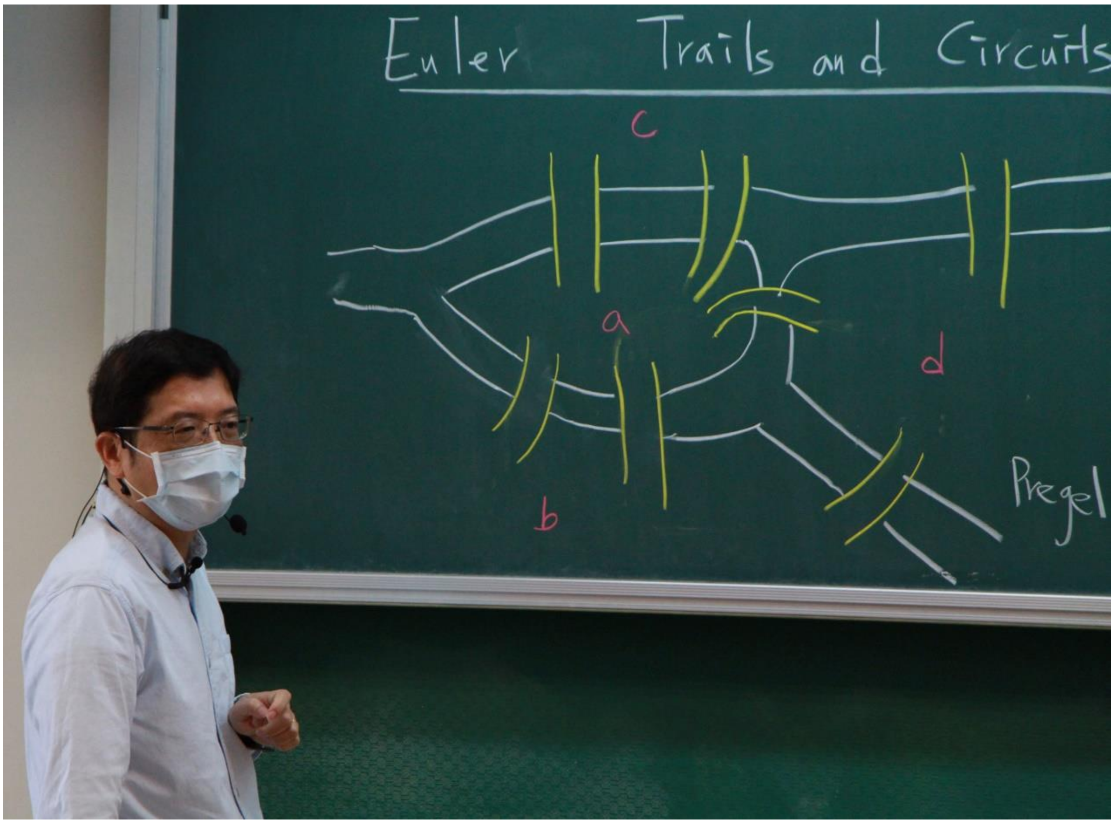
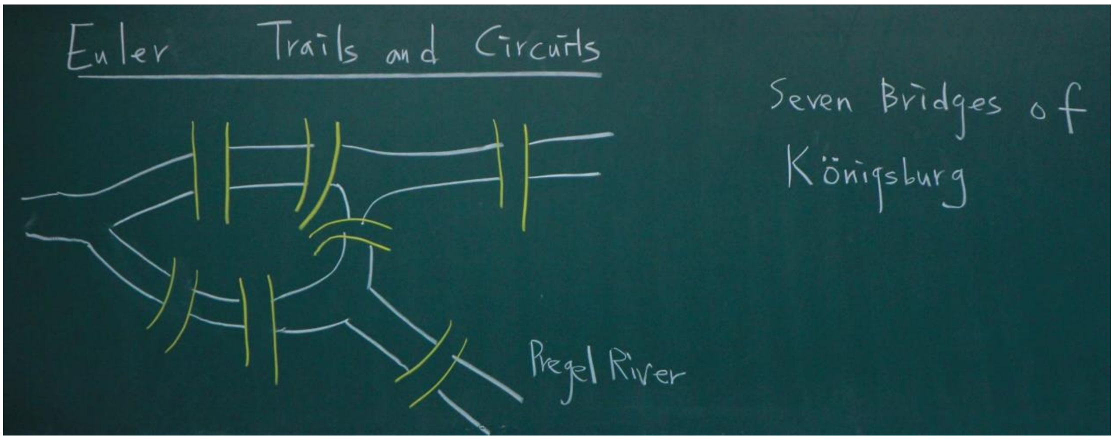
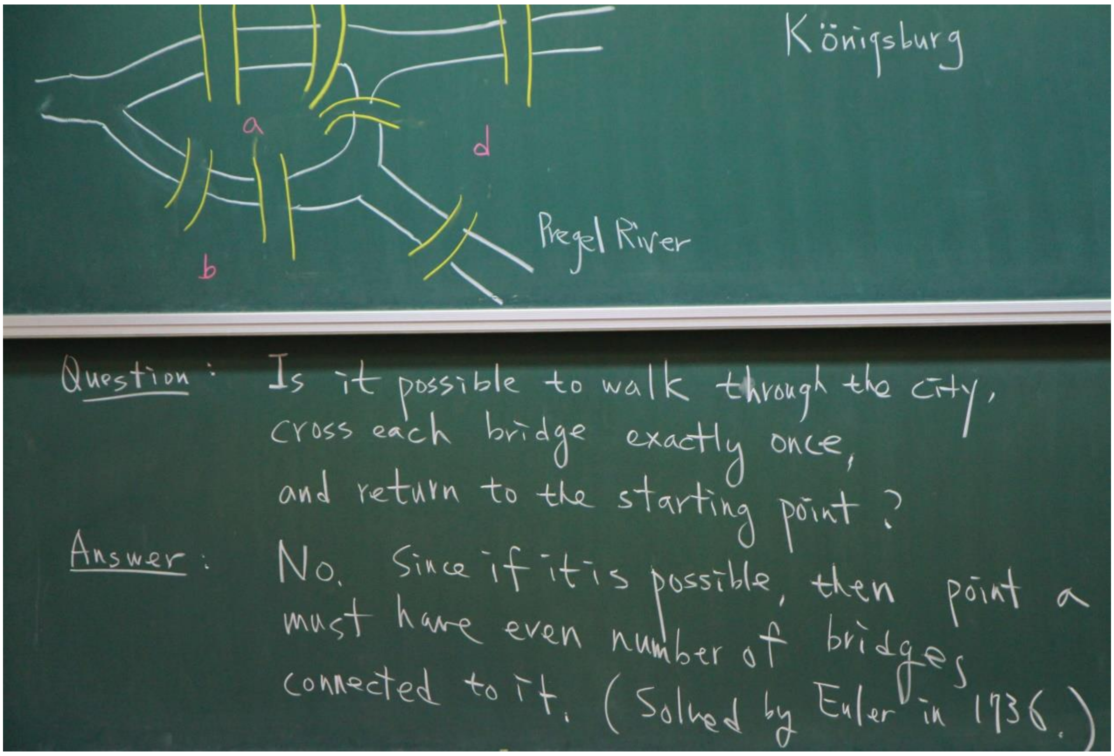
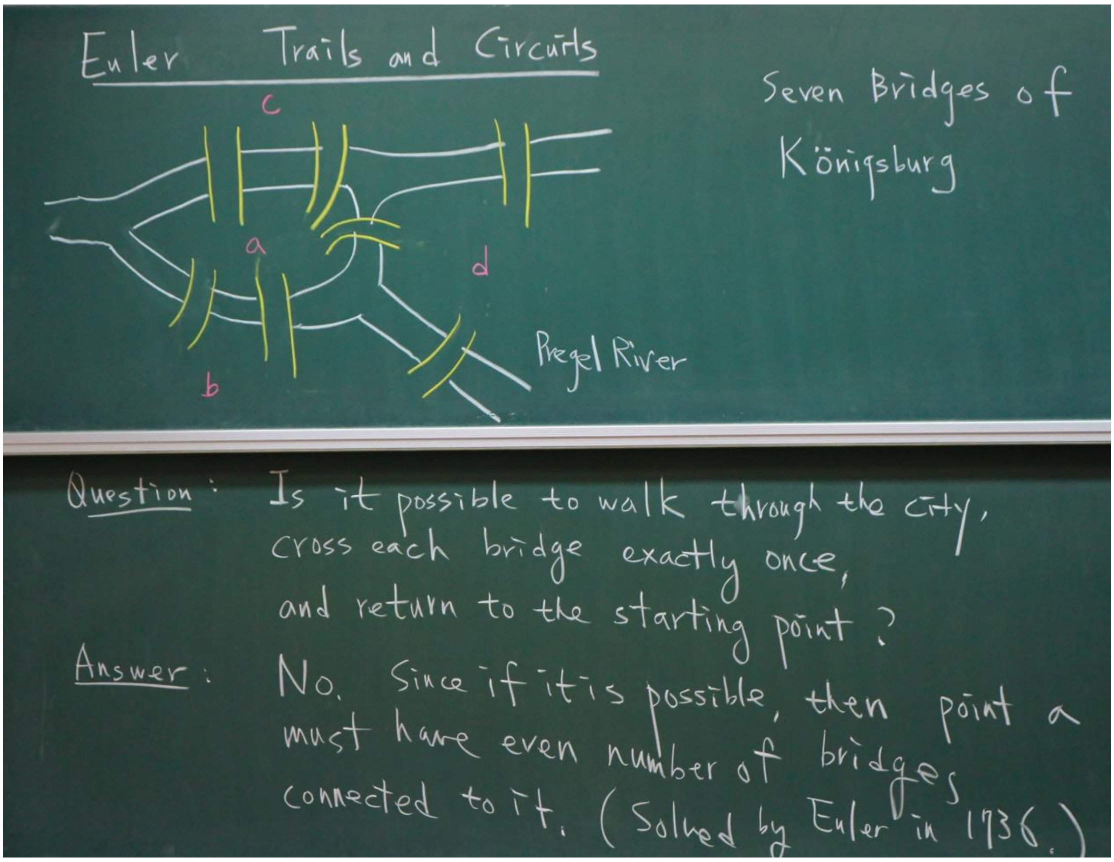
【10920 趙啟超教授離散數學 / 第 20 堂版書】






Königsburg
a
b
d
Pregel River
Question: Is it possible to walk through the city,
cross each bridge exactly once,
and return to the starting point?
Answer: No. Since if it is possible, then point a
must have even number of bridges
connected to it. (Solved by Euler in 1736.)


Euler Trails and Circuits
Seven Bridges of
Königsburg
c
a
b
d
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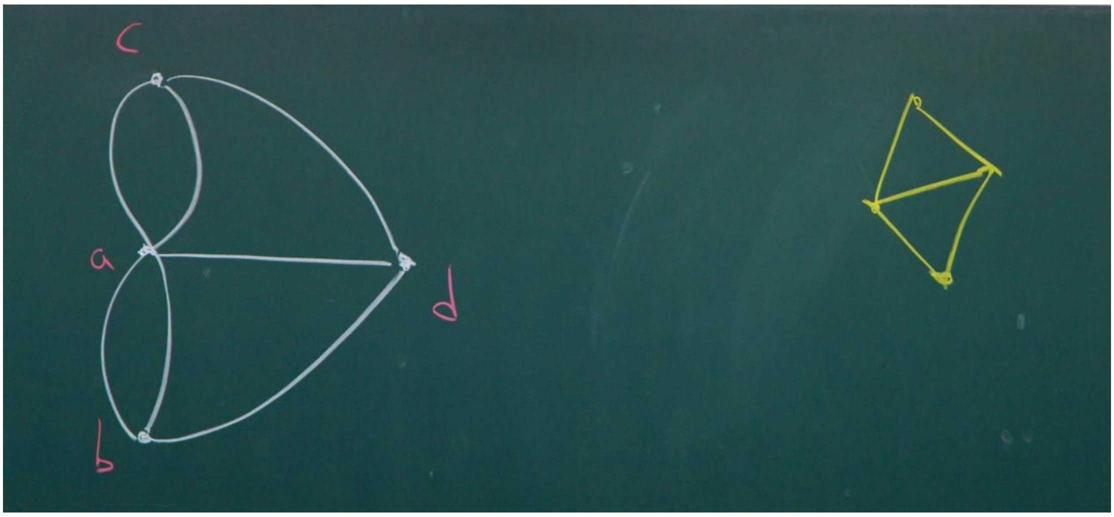
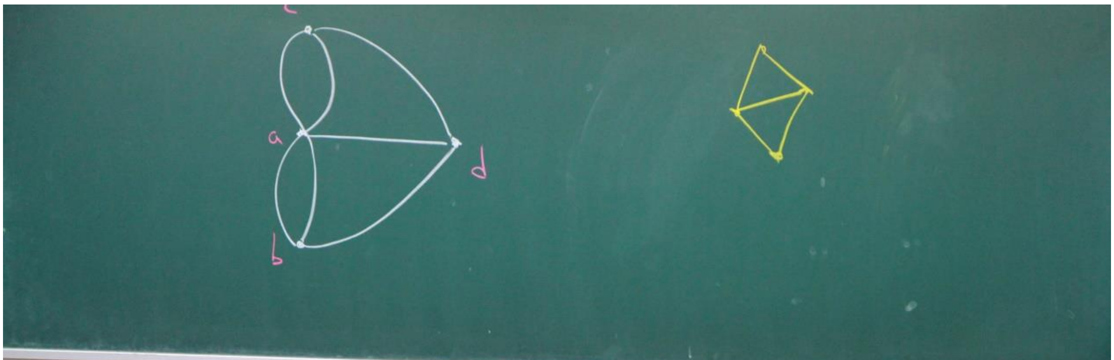
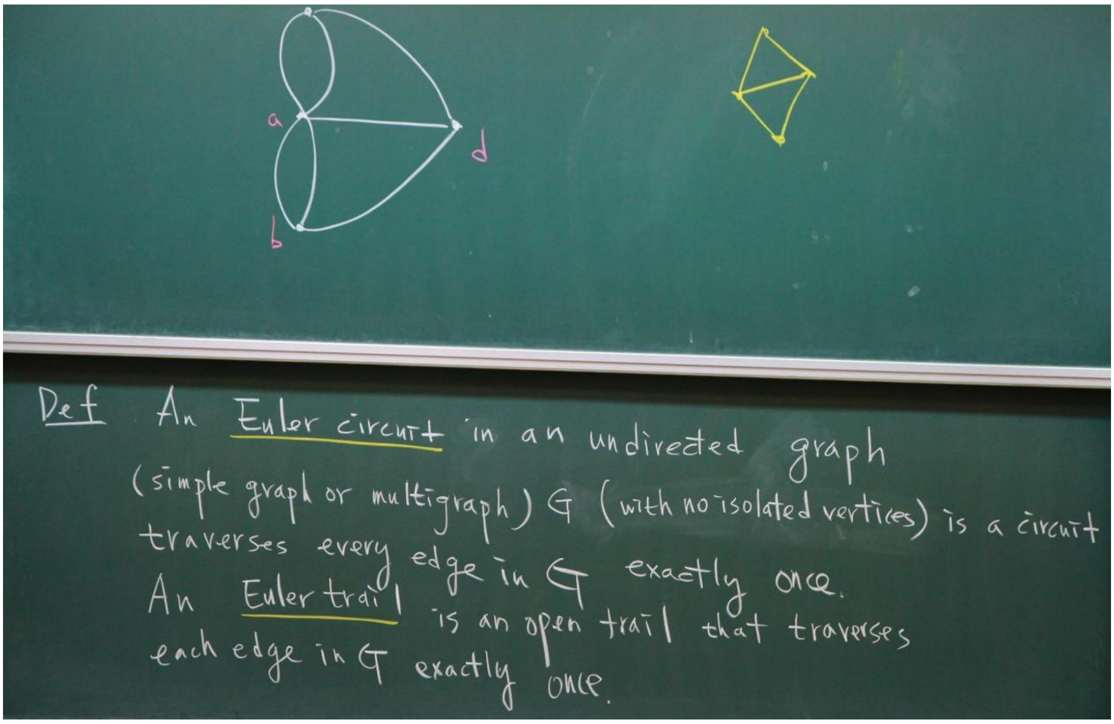




Def An <u>Euler circuit</u> in an undirected graph (simple graph or multigraph) G (with no isolated vertices) is a circuit traverses every edge in G exactly once.

An <u>Euler trail</u> is an open trail that traverses each edge in G exactly once.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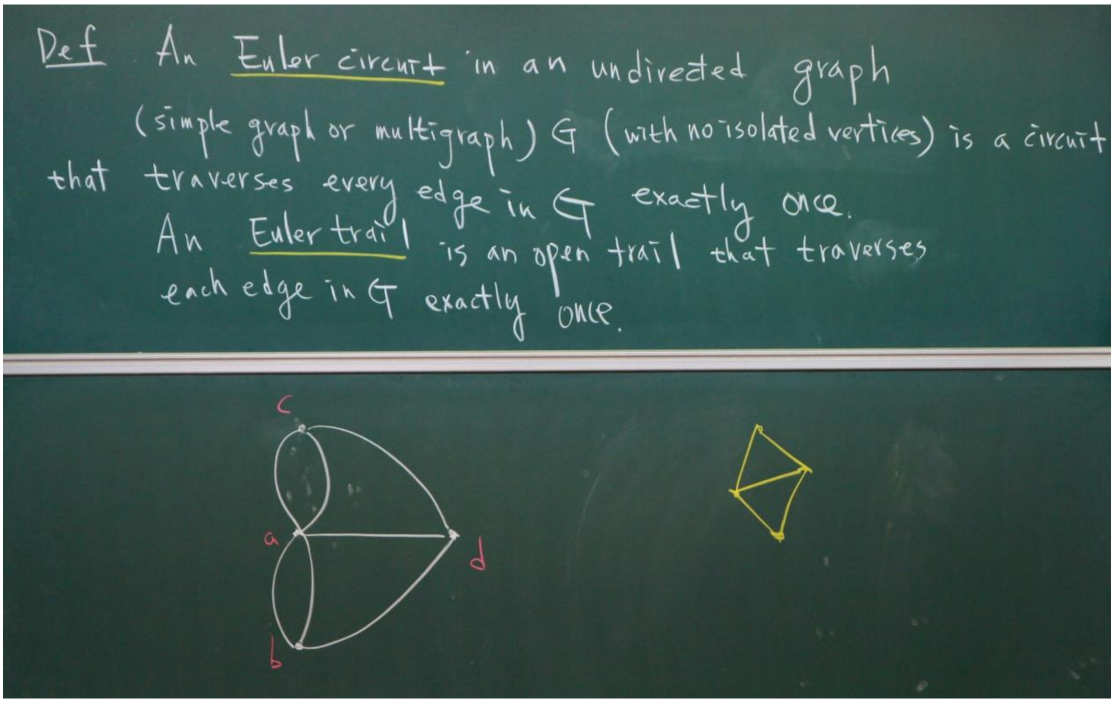
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Theorem G has an Euler circuit iff G is connected and every vertex in G has even degree.

Proof "$\Rightarrow$" (i) For any two distinct vertices in G, since the Euler circuit passes through all edges of G, the Euler circuit connects the two vertices. Hence G is connected.

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(ii) Each time the Euler circuit traverses a vertex v, it passes two edges connected to v: the return edge and the departure edge. Hence the total number of edges at v is twice the number of departure edges and so must be even.

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"$\Longleftarrow$" Let us give an algorithm for finding an Euler circuit.

a. Start at any vertex, say a, and begin traversing along edges at random (never repeating en edge) until we get stuck. Then when we get stuck, we must have

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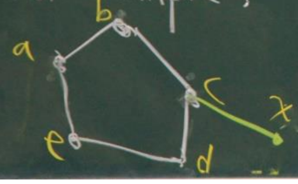
Then when we get stuck, we must have

formed a circuit (since the degree of each vertex is even).

b. If some vertex in the circuit has an unused edge, then "break out".

For example,



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abcdea
breakout:
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An Euler trail is an open trail that traverses each edge in G exactly once.

c. Continue forming circuits and breaking out as long as possible. If breakout is not possible, then we have found an Euler circuit.

Why? Let $e=\{v, w\}$ be an edge in the graph. Since G is connected, there must be a path from a (the starting vertex) to v, say

$a, a_1, a_2, \cdots, v.$

edge. Hence the total number of edges at v is twice the number of departure edges and so must be even.

Then $\{a, a_1\}$ must be in the circuit, or we could break out at a.

Similary, $a_2, \cdots, v$ must all be in the circuit.

Since v is in the circuit, $e=\{v, w\}$ must also be in the circuit, or breakout at v is possible.

Therefore, every edge must be included in the circuit. ∎